



METHOD OF MODAL COMBINATIONS FOR PUSHOVER ANALYSIS OF BUILDINGS

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SUMMARY

Nonlinear static procedures (NSP) are finding widespread use in performance based seismic design since it provides practitioners a relatively simple approach to estimate inelastic structural response measures. However, conventional NSPs using lateral load patterns recommended in FEMA-356 do not adequately represent the effects of varying dynamic characteristics during the inelastic response or the influence of higher modes. To overcome these drawbacks, some improved procedures have recently been proposed by several researchers. A method of modal combinations (MMC) that implicitly accounts for higher mode effects is investigated in this paper. MMC is based on invariant force distributions formed from the factored combination of independent modal contributions. The validity of the procedure is validated by comparing response quantities such as inter-story drift and member ductility demands using other pushover methods and also the results of nonlinear time history analyses. The validation studies are based on evaluation of three existing steel moment frame buildings: two of these structures were instrumented during the Northridge earthquake thereby providing realistic support motions for the time-history predictions. Findings from the investigation indicate that the method of modal combinations provides a basis for estimating the potential contributions of higher modes when determining inter-story drift demands and local component demands in multistory frame buildings subjected to seismic loads.

INTRODUCTION

Current seismic design practice in the United States is still governed by force-based design principles. However, the emergence of performance-based seismic engineering has resulted in increasing use of nonlinear methods to estimate expected seismic demands in a building structure. A widely used and popular approach to establish these demands is a “pushover” analysis in which a mathematical model of the building is subjected to an inverted triangular distribution of lateral forces. While such a load distribution is based on the assumption that the response is primarily in its fundamental mode of vibration, it can lead to incorrect estimates for structures with significant higher mode contributions. This

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accentuates the need for improved procedures that addresses current drawbacks in the lateral load patterns that are used in pushover analyses. Recently, several improved pushover procedures have been proposed [1,2]. These procedures have been shown to provide more accurate estimates of interstory drift values than conventional NSPs using inverted triangular, uniform or other lateral load patterns based on direct modal combination rules suggested in FEMA-356 [3]. The effort required to implement these procedures in routine analysis is significant and, therefore, not an attractive proposition in engineering practice. In order to investigate alternative simple schemes to represent realistic lateral force demands, a new lateral load configuration using factored modal combinations has been recently developed [4]. The accuracy of this approach, to be referred to in this paper as the Method of Modal Combinations (MMC), has been validated on two RC buildings [4]. Other pushover procedures, which are relevant to the work described in this paper, are the methods described by Paret et al. [5] and Sasaki et al. [6].

The principal objective of this paper is to extend the validation study to additional buildings and to compare predictions to other approaches such as the Modal Pushover Analysis (MPA) procedure proposed by Chopra and Goel [2] and to the results of detailed time-history analyses. Three existing buildings with varying story levels (4, 6 and 13) were selected for the evaluation. The six and thirteen story buildings were instrumented during the Northridge earthquake, therefore, recorded base motions were utilized in nonlinear time history (NTH) analyses. For the four-story building, which was not instrumented, three different ground motions were used to predict expected seismic demands.

NONLINEAR STATIC PROCEDURES

The MMC procedure is evaluated by comparing computed interstory drift demands to nonlinear time-history estimates and to other pushover procedures. One set of lateral load patterns was based on recommendations in FEMA-356 while the second methodology considered in the comparative study is the modal pushover analysis (MPA) of Chopra and Goel [2]. A brief overview is presented of the different NSP methodologies used in the study.

Lateral Load Patterns Based on FEMA-356

In FEMA-356, several alternative invariant loading patterns are recommended for estimating equivalent seismic demands. In this study, two loading patterns are considered. These two patterns are permitted when more than 75 percent of the total mass participates in the fundamental vibration mode in the direction under consideration. The following notations are used in this paper to describe these patterns:

NSP-1: The buildings are subjected to a lateral load distributed across the height of the building based on the following formula specified in FEMA-356:

$$F_x = \frac{W_x h_x^k}{\sum_{i=1}^N W_i h_i^k} V \quad (1)$$

where, F_x is the applied lateral force at level 'x', W is the story weight, h is the story height and V is the design base shear, and N is the number of stories. The summation in the denominator is carried through all story levels. This results in an inverted triangular distribution when k is set equal to unity.

NSP-2: A uniform lateral load distribution consisting of forces that are proportional to the story masses at each story level.

Modal Pushover Analysis (MPA)

Modal Pushover Analysis (MPA), developed by Chopra and Goel [2], is essentially the extension of single mode pushover analysis to multi-mode response, and use of the theory of response spectrum analysis to combine the modal contributions. The basic steps of the procedures are as follows:

1. Compute the natural frequencies, ω_n and mode shapes using an elastic model of the system.
2. Run pushover analyses with the loading patterns ($\mathbf{s}_n = \mathbf{m} \phi_n$) based on each mode independently.
3. Idealize each pushover curve as bilinear curves considering negative post-yield stiffness if necessary.
4. Convert the idealized pushover curves into a set of capacity spectrum curves of the corresponding SDOF system using the ADRS conversion from MDOF to SDOF (Note that guidelines provided in ATC-40 [7] capacity spectrum procedure can be used for this purpose).
5. Compute the peak response corresponding to each SDOF system via a nonlinear response history analysis (NRHA) based on an input ground motion for each SDOF system or via inelastic design spectrum.
6. Convert the peak response of SDOF system to the target displacement of MDOF system for each mode separately.
7. From the pushover database (Step 2), extract the peak inelastic response quantities of interest, such as interstory drift and plastic hinge rotations independently for each mode.
8. By using SRSS, determine the combined peak response.

At the first glance, MPA procedure is an adaptation of NRHA for inelastic static analysis. However, this process inherently requires considerable effort except if very few modes are considered in the evaluation. In its original form, MPA is not a static method since it requires repetitive runs of SDOF response history analyses for a given ground motion to identify the target displacement of each mode. Additionally, it requires the use of one or more ground motions unless an inelastic design spectrum is used. Running the pushover analyses independently in each mode and ignoring the contribution of other modes in the formation of plastic hinges is an issue of concern for MPA since it may result in inaccurate estimates of plastic hinge rotation, an important parameter for comparing acceptance criteria in performance-based evaluation.

Method of Modal Combinations (MMC)

In this procedure (see Kunnath [4]) the spatial variation of applied forces is determined from:

$$F_j = \sum \alpha_n \Gamma_n \mathbf{m} \Phi_n S_a(\zeta_n, T_n) \quad (2)$$

where α_n is a modification factor that can assume positive or negative values; Φ_n is the mode shape vector corresponding to mode n ; S_a is the spectral acceleration at the period corresponding to mode n ; and

$$\Gamma = ([\Phi]^T [m] \{t\}) / M_n \text{ in which } M_n = [\Phi]^T [m] [\Phi] \quad (3)$$

If only the first two modes are used in the combination process, then Equation 2 would have the following form:

$$F_j = \alpha_1 \Gamma_1 \mathbf{m} \Phi_1 S_a(\zeta_1, T_1) \pm \alpha_2 \Gamma_2 \mathbf{m} \Phi_2 S_a(\zeta_2, T_2) \quad (4)$$

Therefore, the procedure requires multiple pushover analyses wherein several combinations of modal load patterns are applied. In order to arrive at estimates of deformation and force demands, it is necessary to consider peak demands at each story level and then establish an envelope of demand values for use in performance based-evaluation.

DESCRIPTION OF BUILDINGS

Three special moment resisting steel frame buildings were selected as representative case studies to evaluate the MMC procedure.

4-Story Building

The building was designed in according to 1988 UBC specifications. It is 16.15m in elevation and has a rectangular plan with plan dimensions of 33.27m x 19.2m. The structural system is composed of perimeter MRFs to resist lateral loads and interior gravity frames. The floor plan and elevation view of the building with beam and column sections are shown in Figure 1.

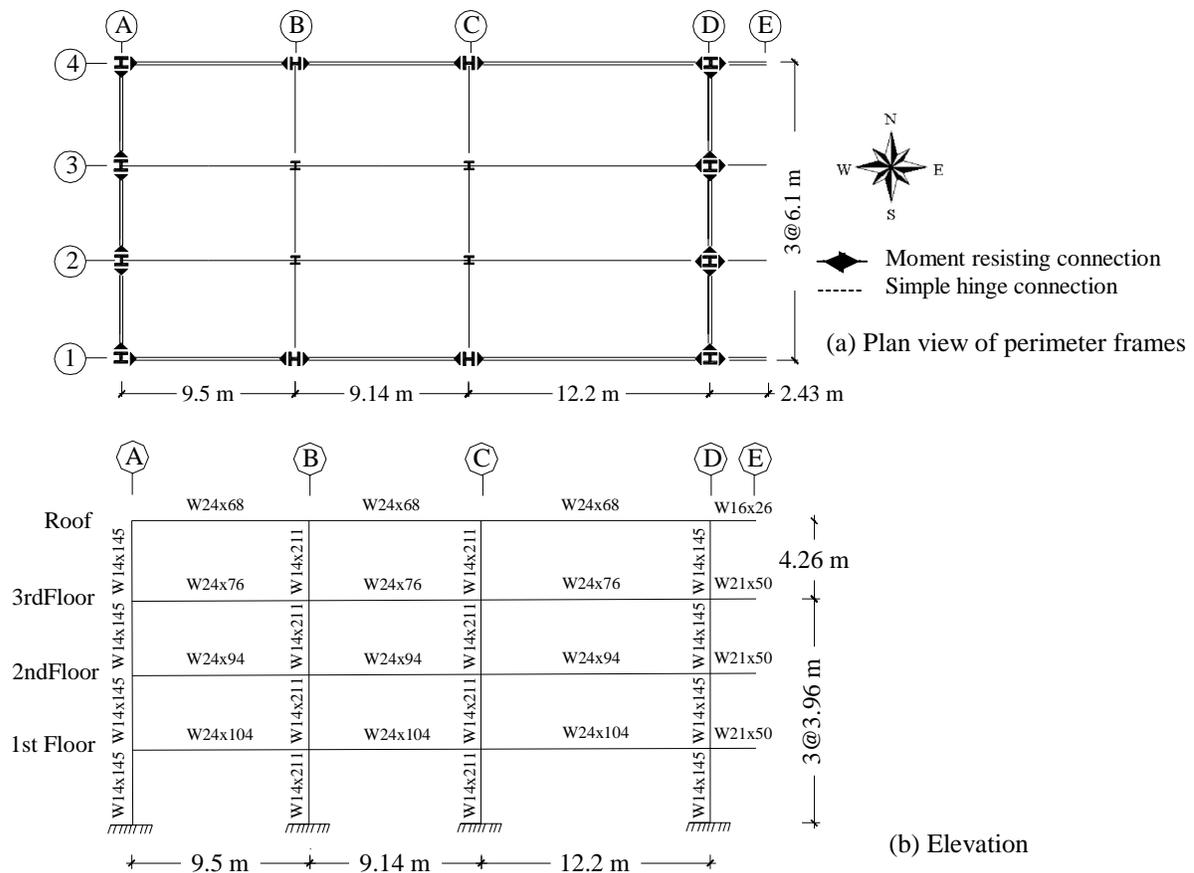


Figure 1. Structural configuration of 4-story building

The columns of the MRFs are embedded into grade beams and anchored to the top of the pile cap, and the foundation system is composed of drilled concrete piers with pile caps, grade beams and tie beams. All columns are made of A-572 grade 50 steel. The girders and beams are made of A-36 steel. The floor system is composed of 15.9 cm thick slab (8.3 cm light weight concrete and 7.6 cm composite metal deck) at all floor levels and the roof. The outside walls are made of thin set brick veneer supported on a metal stud wall. This building suffered significant flange fracture damage in beam-flange to column-flange connections during 1994 Northridge earthquake [8]. All of the severely fractured beam-columns connections were concentrated in the NS direction in the moment frame on Line-D (Fig. 1). No fracture was observed in the NS direction moment frame on Line-A, and only one fracture was observed on Line-1 in the EW direction. This building was not instrumented. Further details of the building are given in Krawinkler et al. [8].

6-Story Building

This moment frame steel structure was designed in accordance with 1973 UBC requirements. The rectangular plan of the building measures 36.6m x 36.6m with a 8.2cm thick light weight concrete slab over 7.5cm metal decking. The primary lateral load resisting system is a moment frame around the perimeter of the building. Interior frames are designed to carry only gravity loads. The plan view and the elevation of a typical frame are shown in Figure 2. The building was instrumented with a total of 13 strong motion sensors at the ground, 2nd, 3rd and roof levels. The building performed well in the Northridge earthquake with no visible signs of damage. In constructing the building model, the columns were assumed to be fixed at the base level (all columns are supported by base plates anchored on foundation beams which in turn are supported on a pair of 9.75m - 0.75m diameter concrete piles). The specified minimum concrete compressive strength at 28 days was 20.7 MPa. Section properties were computed for A-36 steel with an assumed yield stress of 303 MPa. The total building weight (excluding live loads) was estimated to be approximately 34,644kN. Additional details including calibration of the building model is reported in Kunnath et al. [9].

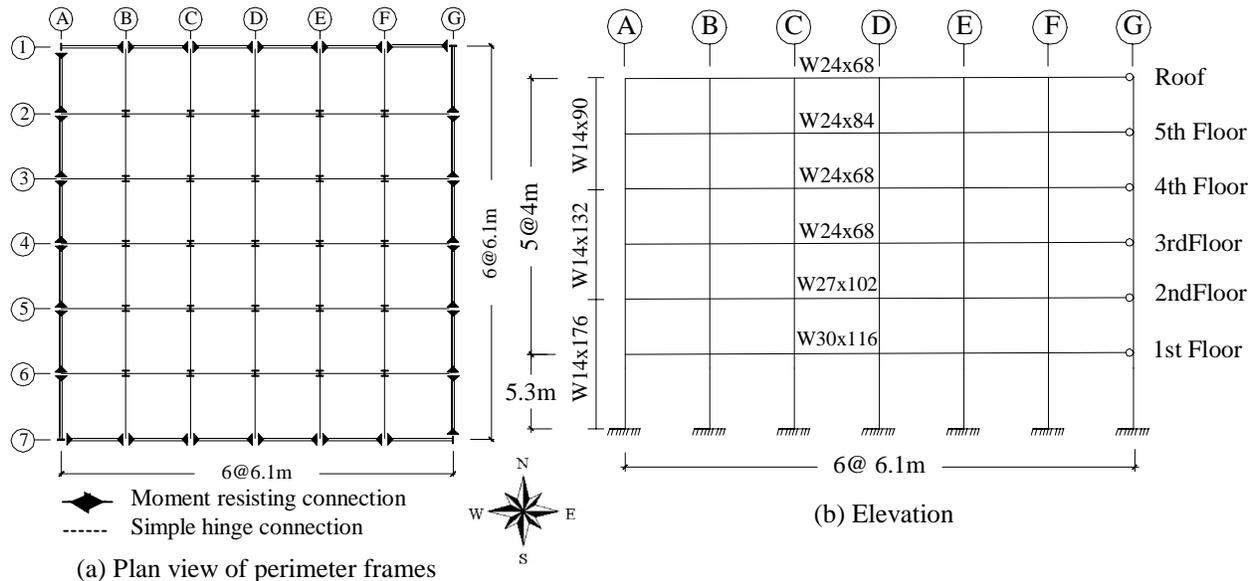


Figure 2. Structural configuration of Burbank 6-story building

13-Story Building

This building is located in South San Fernando Valley about 5 km southwest of the Northridge epicenter and is composed of one basement floor and 13 floors above ground. Built in accordance with the 1973 UBC code, this structure has been the subject of a previous investigation [9,10]. As shown in Figure 3, it has a 48.8m square plan and 57.5m in elevation. The lateral load resisting system is composed of four identical perimeter frames. The floor plan increases at the second floor to form a plaza level that terminates on three sides into the hillside thereby making this level almost fixed against translation. Recorded response of the building during the Northridge earthquake indicates a peak horizontal acceleration of 0.44g, 0.32g and 0.33g at the ground, 6th floor and roof levels. Weld fracture damage was observed primarily in the NS direction.

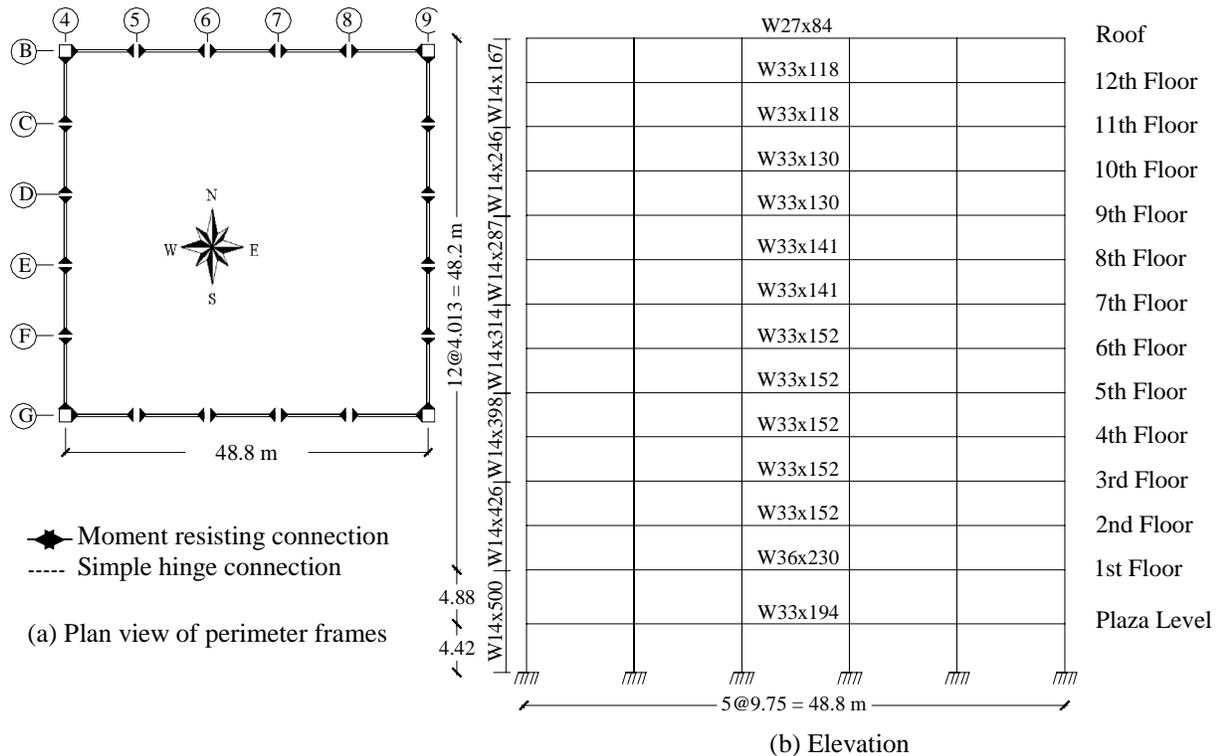


Figure 3. Structural configuration of the 13-story building

SUMMARY OF EVALUATION

The evaluation process consisted of comparing the computed demands using MMC with time-history analysis and with other pushover methods. For the two instrumented buildings, the recorded base motions served as the input accelerations for the time history analyses. Since the actual ground motions did not produce significant inelastic behavior, the records were scaled so as to induce a peak interstory drift of approximately 2 percent at any level. The target displacements for the pushover procedures were then based on the peak time-history induced story drifts. This approach provides a rational basis for comparing the demands obtained with different methods. Since instrumented information was not available for the four-story structure, three ground motion records were selected from the recommended set in ATC-40 [7]. Comparison of interstory drift demands comprised the primary basis for the evaluation. Typical member ductility demands (based on plastic rotations) were also evaluated for the MMC and FEMA procedures.

The MPA procedure does not lend itself to a direct evaluation of component ductility. Details of the ground motions used for the time-history analyses of the three buildings are presented in Table 1.

Table 1. Details of ground motions used in nonlinear time history analyses

Year	Earthquake	Recording Station	PGA (g)	EQ. Scale Factor *		
				4-Story	6-Story	13-Story
1989	Loma Prieta	Hollister, South & Pine	0.370	1.0	-	-
1989	Loma Prieta	Gilroy #2	0.350	2.1	-	-
1994	Northridge	Moorpark	0.290	4.0	-	-
1994	Northridge	Burbank	0.299	-	2.3	-
1994	Northridge	Woodland Hills	0.318	-	-	1.9

* Scale factor used to achieve the target interstory drift ratio of two percent

Analytical Modeling

The nonlinear evaluations were carried out using the open source finite element framework, OpenSees [11]. A nonlinear beam-column element that utilizes a layered ‘*fiber*’ section is used to model all components in the frame models since the interaction of axial force and flexure is automatically incorporated. The element is based on a force formulation that considers the spread of plasticity. Since the objective of the evaluation is to evaluate various pushover procedures rather than simulate local connection fracture, the modeling of the members and connections was based on the assumption of stable hysteresis derived from a bilinear stress-strain model. Since the buildings are symmetric in plan, only two-dimensional models of a single frame were developed for each building. In the case of the four-story building, the exterior frame along EW direction (Line-1) was modeled. Similarly, frame models for the six and thirteen story buildings were developed for the exterior frames in EW direction. The elastic models were validated using available recorded data and typical simulations are displayed in Figure 4.

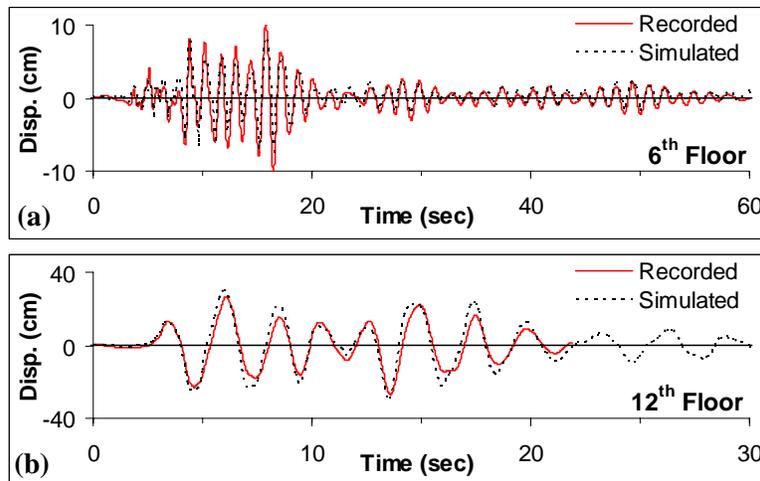


Figure 4. Validation of analytical models: comparison of recorded and computed response (a) EW response at roof level of 6-story building; (b) EW response at roof level of 13-story building

Evaluation of Method of Modal Combination (MMC)

The validation exercise presented in Figure 4 was obtained using elastic models indicating minimal or no inelastic behavior. Hence, it was necessary to scale the recorded ground motions so that interstory drifts reached a magnitude to cause yielding in elements and provide a more reasonable basis to evaluate the adequacy of the pushover methods. The scale factors used to produce 2 percent peak story drifts are given in Table 1 for each ground motion. The 5-percent damped elastic acceleration and displacement response spectra computed from the amplified motions are presented in Figure 5. Also marked on these figures are the first two modal periods for the four- and six-story models and three modal periods for the thirteen-story model.

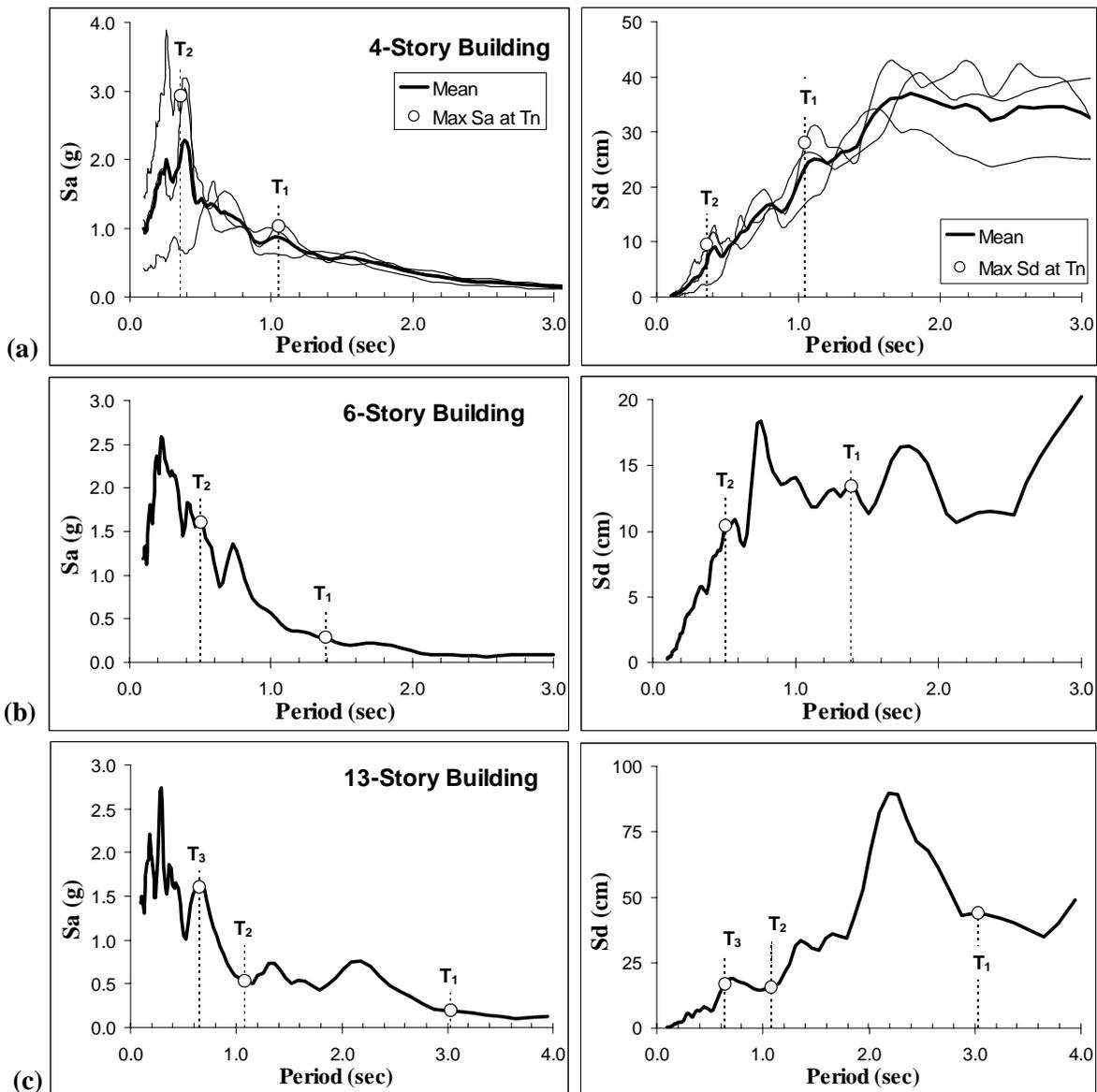


Figure 5. 5-percent damped pseudo acceleration spectra and displacement spectra for (a) 4-story building (based on three ground motions); (b) 6-story building; (c) 13-story building

Evaluation of Interstory Drift Demands

As indicated previously, three earthquake records were used for the nonlinear time history (NTH) analysis of the four-story building. The MMC response is based on the envelope of demands resulting from Mode $1 \pm$ Mode 2 (using peak S_a of the three ground motions). The resulting lateral forces using such a modal combination is shown in Figure 6. Plots of the displacement and drift profile for both NTH runs and the various pushover methods are shown in Figure 7. The peak displacement profiles are generally similar for all methods. The variation of interstory drift indicates that both MPA and MMC capture the demands with reasonable accuracy though the demand at the first story is slightly over-estimated. Of the two FEMA methods, NSP-1 (first mode distribution) is a better indicator of seismic demands though the drifts at the uppermost level are under-estimated. Note that NTH gives the highest demand at third story that implies some contribution of the second mode in the response.

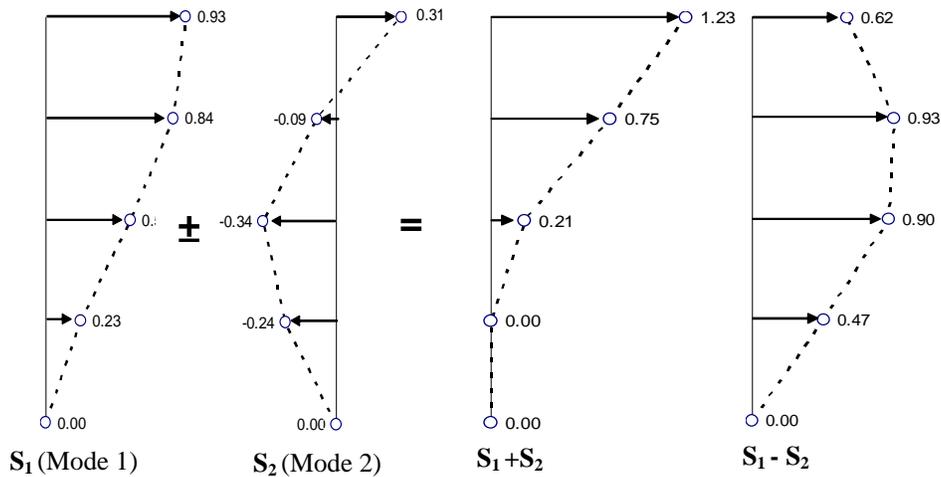


Figure 6. Spatial distribution of lateral forces for 4-story building, $S_n = \Gamma_n \mathbf{m} \phi_n$, for $n = 1$ and 2

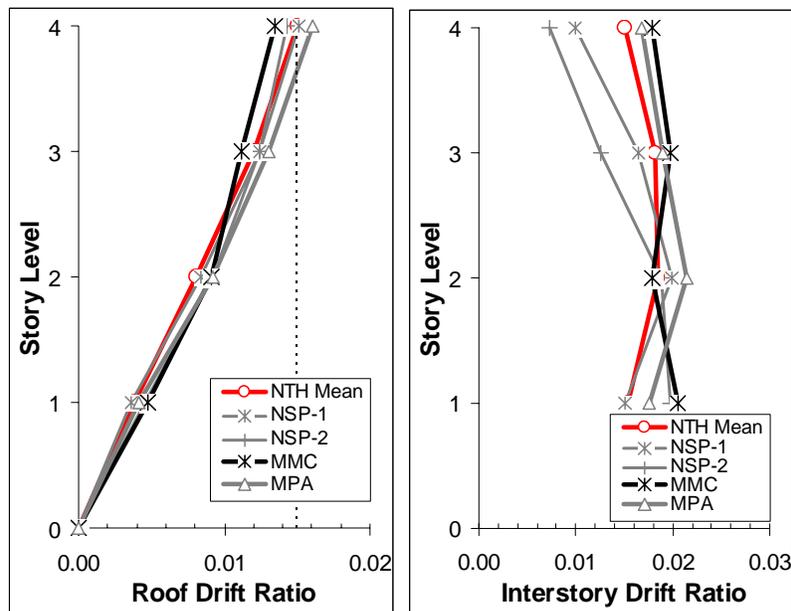


Figure 7. Comparison of roof drift and interstory drift ratio using various methods for 4-story building

As in the case of the four-story building, the MMC procedure was developed from lateral forces using a modal combination based on Mode 1 \pm Mode 2. This combination led to the lateral force distribution as indicated in Figure 8. As is evident from these distributions, the two combinations place increased demands on either the upper or middle stories and is a function of both the mode shape and the spectral demands at these modal periods. The resulting story demands are plotted in Figure 9 along with demand estimates from the other methods. Significant higher mode effects are apparent in Figure 9b. MMC captures the highest story drift as well as the other story drifts reasonably, however the drift demand is overestimated at the first story. The MPA procedure generally under-predicts the demands at most levels. For this particular building none of the FEMA procedures show good correlation with the time history results in terms of the story drifts, they overestimate the demand at the lower levels and underestimate it at upper levels.

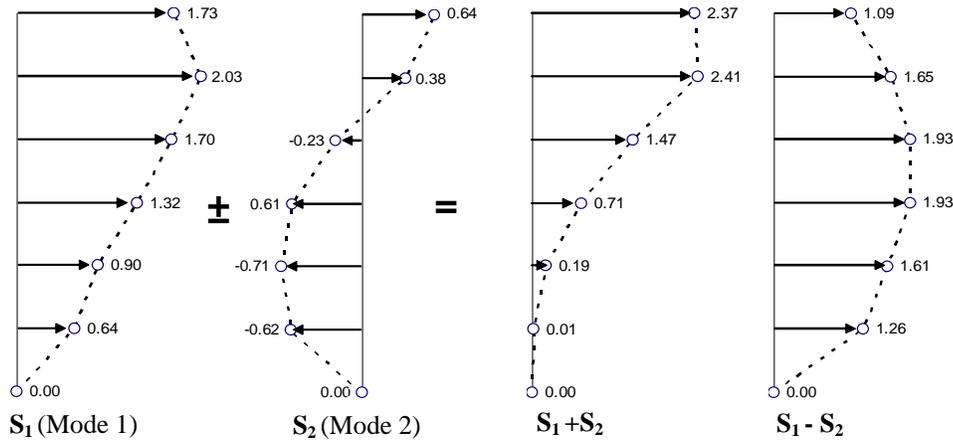


Figure 8. Spatial distribution of lateral forces for 6-story building, $S_n = \Gamma_n \mathbf{m} \phi_n$, for $n = 1$ and 2

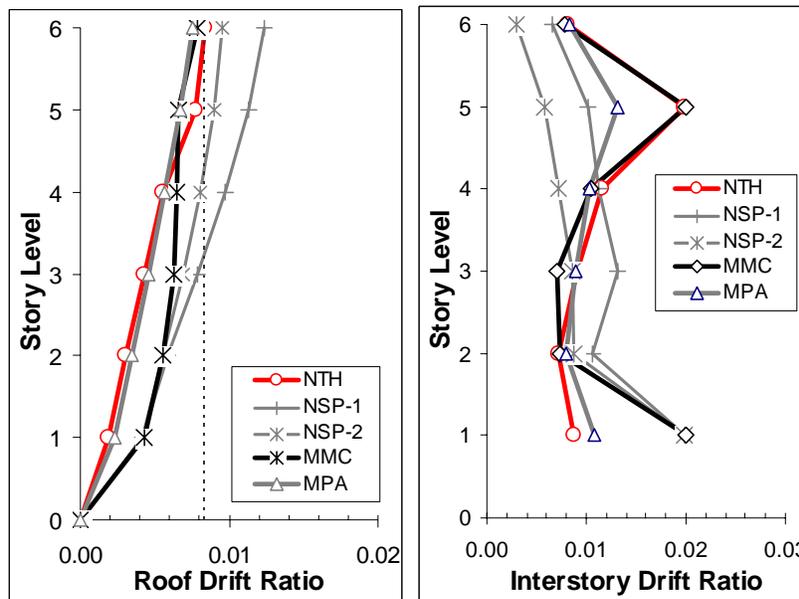


Figure 9. Comparison of roof and interstory drift ratios using various methods for 6-story building

For the thirteen-story building, the MMC procedure was developed from lateral forces using a modal combination based on Mode 1 \pm Mode 2 + Mode 3. This combination led to the lateral force distribution as indicated in Figure 10. Inclusion of third mode indicates a significantly altered load distribution. For this building, the combinations based on the first two modes, as used for the other buildings, was also evaluated in addition to the two combinations shown in Figure 10, and analyses were conducted for two configurations separately. The resultant demands are given in Figure 11 for comparison. The predicted demands from the other methods are also presented. In general demands predicted by MMC are in agreement with the computed demand from NTH analysis. Incorporation of the third mode improved the capability of MMC to capture the time-history demands. Except for the lower story levels, neither FEMA procedures nor MPA show good correlation with the computed demand from NTH analysis.

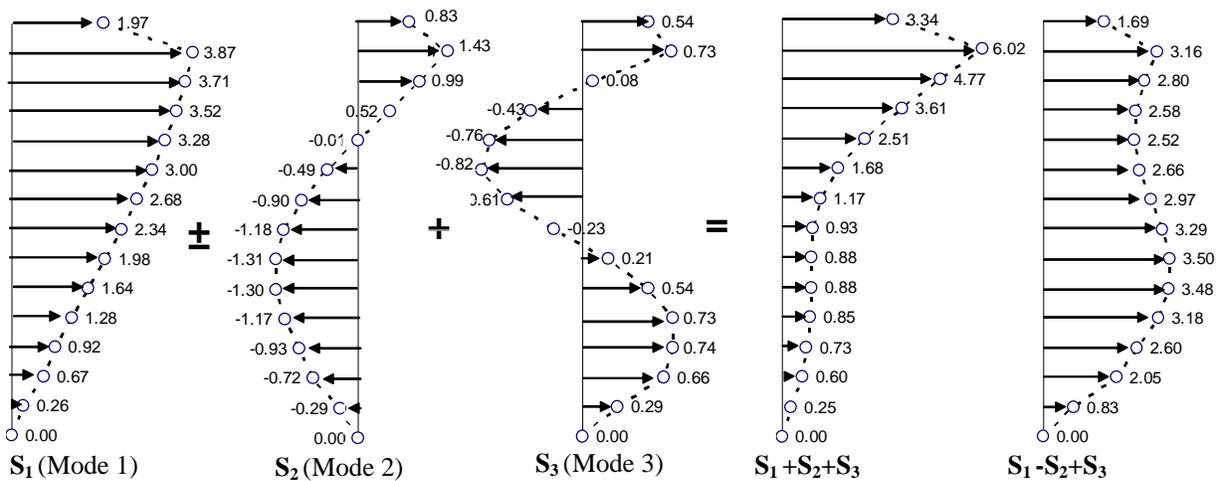


Figure 10. Spatial distribution of lateral forces for 13-story building, $S_n = \Gamma_n \mathbf{m} \phi_n$, for $n = 1, 2$ and 3

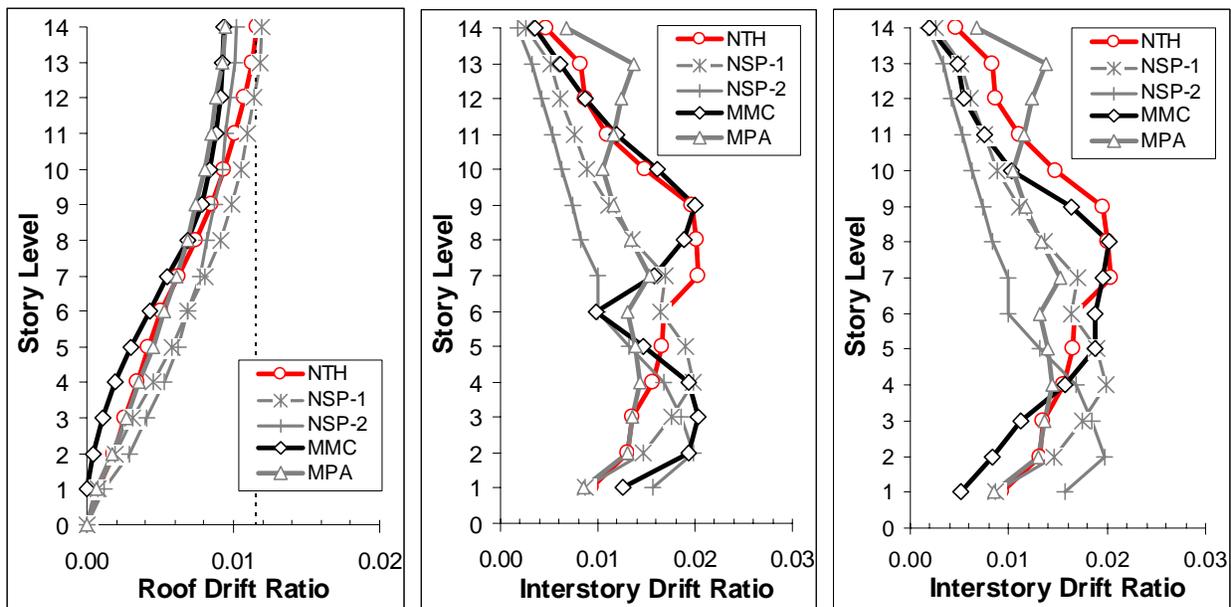


Figure 11. Comparison of roof and interstory drift ratios using various methods for 13-story building

The displacement profile of the six and thirteen story buildings during the time history analysis is captured in Figures 12 and 13. Also shown in these figures are the corresponding story drift histories. Though not immediately evident from the figure, it was observed that the drift profile is initially representative of the modal contributions to the response based on spectral demands and that these demands change as the systems become inelastic and the modal periods shift along with the corresponding spectral demands.

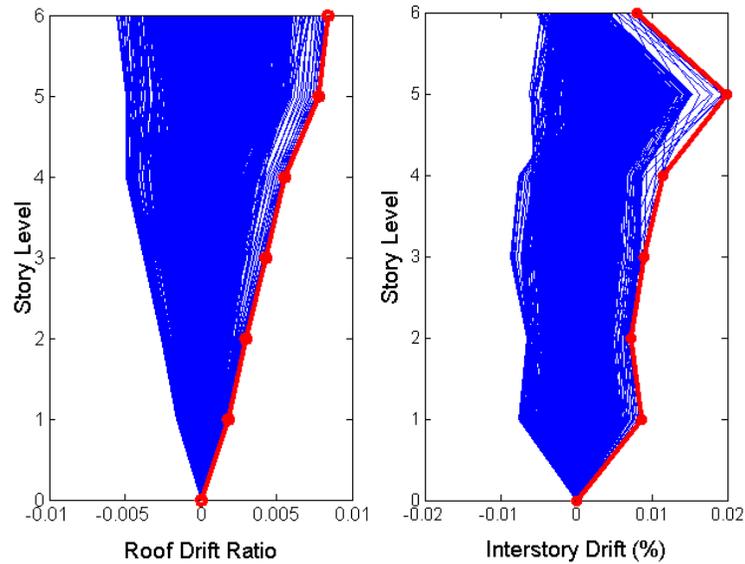


Figure 12. Profile of response history for the six-story structure:
(a) Roof drift ratio history; (b) Interstory drift ratio history

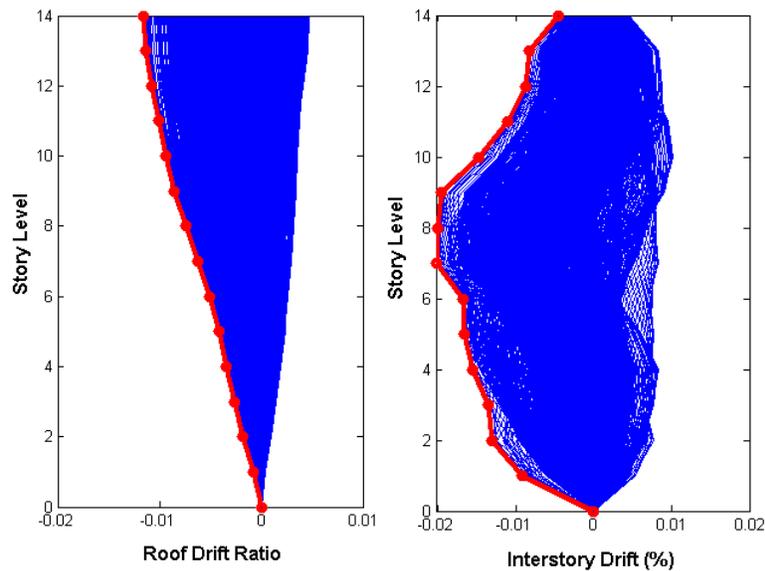


Figure 13. Profile of response history for the thirteen-story structure;
(a) Roof drift ratio history; (b) Interstory drift ratio history

Evaluation of Component Ductility Demands

Another important parameter in seismic response analysis is the estimate of ductility demands in individual components. FEMA-356, for example, uses component acceptance criteria using ductility demands as the fundamental basis of its performance-based evaluation methodology. In this section, the effectiveness of the MMC procedure to estimate component demands is investigated. Tables 2 and 3 show typical ductility demands for column elements at critical story levels in the six and thirteen story buildings experiencing the highest deformation demand. Also given are the global system ductility demands which are less than the observed local story and component ductility demands. Similar results were obtained in a more comprehensive study by the authors examining ductility demands of RC and steel buildings [12]. These results serve as evidence that designing a building to achieve a certain ductility demand may result in much larger demands at the local level.

Comparisons of the ductility demand from pushover procedures with by nonlinear time-history analyses show that the predicted demands are remarkably similar to those estimated. A more visual comparison is provided in Figures 14 and 15 where the moment-rotation behavior of three typical column sections undergoing inelastic deformation is displayed. While cumulative effects cannot directly be incorporated into any static procedure, the ability of MMC to estimate component deformations is clearly demonstrated in these figures.

Table 2. Typical ductility demands in 6-story building

	Location	NSP-1 *	NSP-2 *	NTH	MMC
Global	-	1.53		-	1.92
5th Story	-	0.0	0.0	-	2.02
5th Story Column	Interior	0.0	0.0	2.81	2.73

* NSP-1: Inverted triangle; NSP-2: Mass proportional

Table 3. Typical ductility demands in 13-story building

	Location	NSP-1 *	NSP-2 *	NTH	MMC
Global	-	2.08	2.24	-	2.05
7th Story	-	2.19	1.32	-	2.59
7th Story Column	Interior	3.28	1.67	3.69	3.74
9th Story	-	1.30	0.0	-	1.90
9th Story Column	Interior	1.61	0.0	2.60	2.70

* NSP-1: Inverted triangle; NSP-2: Mass proportional

CONCLUSIONS

The popularity of nonlinear static pushover analysis in engineering practice calls into question the validity of conventional lateral load patterns used to estimate inelastic demands. The aim of the present work is to develop alternative multi-mode pushover analysis procedures by indirectly accounting for higher mode contributions but yet retaining the simplicity of invariant distributions in a theoretically consistent manner. A new combination scheme is investigated in this paper and compared to both time-history procedures and other pushover methods. The evaluation is based on a series of analyses of existing steel moment frame buildings.

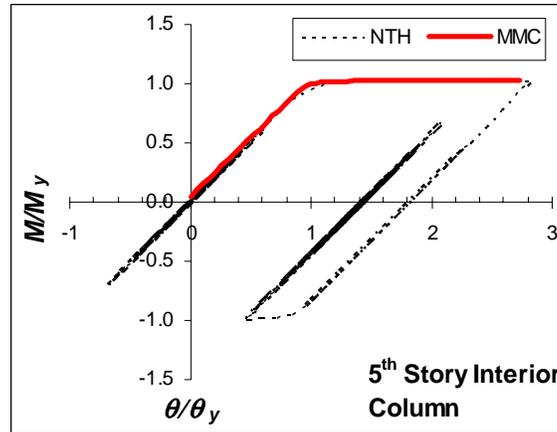


Figure 14. Comparison of ductility demands in typical interior column from MMC and NTH for 6-story building (θ implies total rotation, θ_y implies yield rotation)

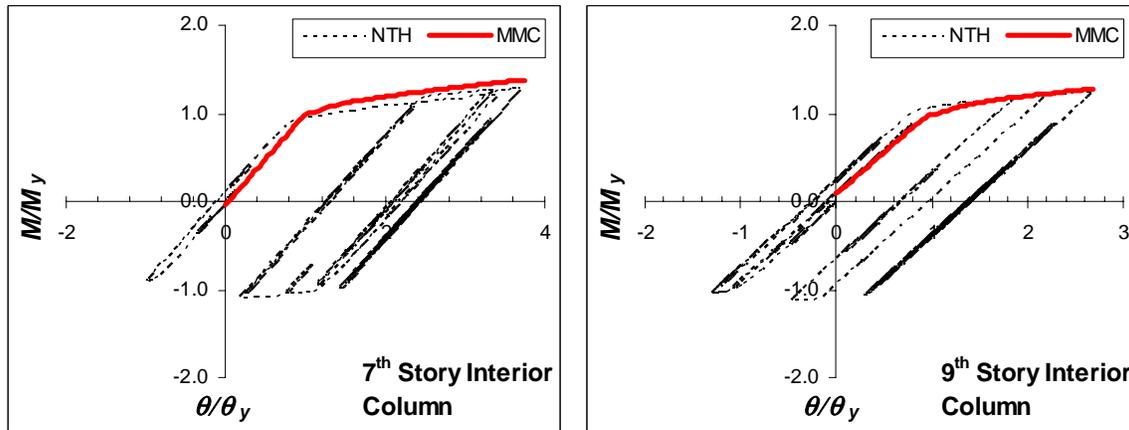


Figure 15. Comparison of inelastic static demand using MMC and cyclic demand in typical interior column of the 13-story building

This study indicates that pushover methods utilizing lateral force distributions based on a single mode are not capable of predicting the story level at which the critical demands occur. On the other hand, the results of modal combination procedures based on ongoing research appears to be promising in terms of better estimating peak values of critical inelastic response quantities such as inter-story drifts and plastic hinge rotations. It is shown that considering sufficient number of modes, interstory drifts estimated by MMC is generally similar to trends noted from NTH analyses unless the building is deformed far into the inelastic range with significant strength and stiffness deterioration.

Higher mode effects on seismic demand are dependent both on the frequency content of the ground motion and the characteristics of the structural system even for regular low-rise buildings (based on findings from the four-story building evaluation). In the present phase of the research, the force distributions are based on modal contributions in the elastic state of the system. The influence of higher modes in the inelastic phase of the response can be incorporated by introducing modification factors that

account for changes in spectral demands due to inelastic effects. Additional studies considering various structural systems and ground motions are ongoing to further validate the methodology.

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